MATH 323.001
Dr. Henjin Chi

Test 1

Name Key
2/27/2006

I. Truth/False Questions (10 points, 2 points each)

F 1. The inversion of a straight line outside a circle C is a circle passing through the center of C and tangent to C.

F 2. If a convex cyclic quadrilateral ABCD is constructed so that $\overline{AC}$ is a diameter of the circle, then ABCD is a parallelogram.

T 3. The center of nine-point circle of a triangle is the midpoint of line segment between orthocenter and circumcenter of the given triangle.

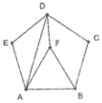
F 4. Two circles are said to be orthogonal if their tangents are perpendicular to each other.

F 5. In polar coordinate, (-5, 60) is in third quadrant.

II. Short Answer Questions (20 points)

1. Points A(2,-2), B(8,-2) and C(6,5) are three vertices of an isosceles trapezoid. Find the coordinate of D if $\overline{AB} \parallel \overline{CD}$. (2 points)

Answer: (4,5)

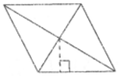


2. In the figure, ABCDE is a regular pentagon, and ABF is an equilateral triangle. What is the measure of angle ADF? (3 pts)

Answer: 18° or $\frac{\pi}{10}$

3. A patio is made up of four circles, all of radius 8 feet, and the region in the middle. What is the total area of the patio? (3 pts)

Answer: $256 + 192\pi = 859\ ft^2$ 859.185789



4. A rhombus has diagonals of length 60 cm and 80 cm. From the point of intersection of the diagonals, a segment is drawn perpendicular to one of the sides of the rhombus. What is the length of that segment? (3 pts)

Answer: 24 cm

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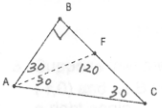
5. Suppose that circle C has its center at point O. Point P, outside circle C, has shortest distance 8 cm to C and longest distance 18 cm to C. (4 pts)

   a. What is the Power(P,C)? __144__________

   b. What is the Power(O,C)? __-25__________

6. Convert rectangular coordinate (-5, 12) to polar coordinate. (2 pts)

   Answer: __(13, 1.96558) or (13, 112.6198°)__

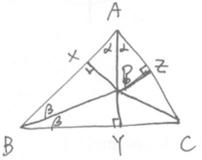


7. When the triangular piece of paper ABC is folded along the dotted line AF, point B lands on side AC. We know AB is 1 unit and AC is 2 units. If the measure of angle B is 90°, What is the measure of angle AFC? (3 points)

   Answer: __120°________

III. Prove (20 points, 5 points each question)

1. Prove that the three angle bisectors of a triangle are concurrent.



Given $\triangle ABC$, AP is angle bisector of $\angle BAC$ and BP is an angle bisector of $\angle ABC$. The two angle bisector intersect at P.

Construct $\overline{PX} \perp \overline{AB}$, $\overline{PY} \perp \overline{BC}$, and $\overline{PZ} \perp \overline{AC}$

$\overline{AP} = \overline{AP}$, $m\angle PXA = m\angle PZA = 90°$, and $\angle XAP \cong \angle ZAP$ (angle bisector)

$\therefore m\angle XPA \cong m\angle ZPA = 90 - \alpha$

Therefore, $\triangle APX \cong \triangle APZ$ (ASA). $\therefore \overline{PX} = \overline{PZ}$

In $\triangle BPX$ and $\triangle BPY$, we have $\overline{BP} = \overline{BP}$, $\angle PBX \cong \angle PBY$ (Angle bisector) and $\angle PXB \cong \angle PYB$ are right angle so $\angle XPB \cong \angle YPB$ $\therefore \triangle BPX \cong \triangle BPY$

$\therefore \overline{PX} = \overline{PY}$. From above, we have $\overline{PX} = \overline{PY} = \overline{PZ}$. and $\overline{CP} = \overline{CP}$

$\angle PYC \cong \angle PZC$ $\therefore \triangle PZC \cong \triangle PYC$ (RHS). Hence $\angle PCZ \cong \angle PCY$

$\therefore$ CP is an angle bisector of $\angle ACB$. All 3 angle bisector intersect at P. So they are concurrent.

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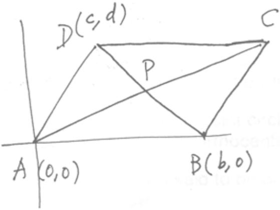
2. Use rectangular coordinate system to prove that the diagonals of a parallelogram bisect each other. Pick P as midpoint of $\overline{AC}$



The intersection point P has coordinates $\left(\frac{b+c}{2}, \frac{d}{2}\right)$

$AP = \sqrt{\left(\frac{b+c}{2}\right)^2 + \left(\frac{d}{2}\right)^2} = PC$

$DP = \sqrt{\left(\frac{c-b}{2}\right)^2 + \left(\frac{d}{2}\right)^2}$

$BP = \sqrt{\left(\frac{c-b}{2}\right)^2 + \left(\frac{d}{2}\right)^2}$

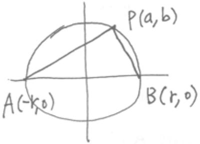
$\therefore$ AP = PC and DP = BP

slope of $DP = \frac{d}{c-b}$ and slope of $BP = \frac{d}{c-b}$

So D, P, B is a straight line.

QED

3. Suppose that the point P(a,b) is on circle $x^2 + y^2 = r^2$. Let A be the point (-r,0) and let B be point (r,0). Use coordinates to show that angle APB is a right angle.



slope of AP: $\frac{b}{a+r}$

slope of BP: $\frac{b}{a-r}$

P(a, b) is on the circle so $a^2+b^2=r^2$

The product of the slope $\left(\frac{b}{a+r}\right) \cdot \left(\frac{b}{a-r}\right)$

$= \frac{b^2}{a^2-r^2} = \frac{b^2}{-b^2} = -1$

$\therefore \angle APB$ is a right angle

QED

4. Complete the truth table below.

P	Q	P→Q	Q→P	P or (not Q)
True	True	True	True	True
False	True	True	False	False
True	False	False	True	True
False	False	True	True	True